# Mathematical Description of Slope Evolution

Pierce and Colman (1986) – Paper available on website – present an exact solution for a fault scarp in a flat plain: no background slope ($b$ = 0); initial condition = scarp angle $\alpha$ (not a vertical scarp, usually the angle of respose, ca. 33º). They solve for diffusivity, $K$, in terms of scarp age ($t$) and morphologic measurement of the maximum scarp gradient, $\tan\theta_m$, for a scarp of total height ($h$) (not to be confused with the amplitude = ½ height used by Hanks et al):

$$K_c = \left[ \frac{h}{4t^{1/2} erf^{-1}\left(\tan\theta_m / \tan\alpha\right)\tan\alpha} \right]^2$$

Solve for maximum slope as a function of time:

$$\tan\theta_m = (\tan\alpha) erf\left[ \frac{h}{4(K_c t)^{1/2}\tan\alpha} \right]$$

To correct this relationship for application to a triangular ridge (closer approximation to a cinder cone), I tested it against a numerical solution for the triangular ridge and found the ratio between $\tan\theta_m$ of the ridge and of the scarp had the form of the error function. With this clue, I found the following provided a very good approximation for the evolution of the triangular ridge:

$$\tan\theta_m = (\tan\alpha) erf\left[ \frac{h}{4(K_c t)^{1/2}\tan\alpha} \right] \sqrt{erf\left[ \frac{h}{4(K_c t)^{1/2}\tan\alpha} \right]}$$

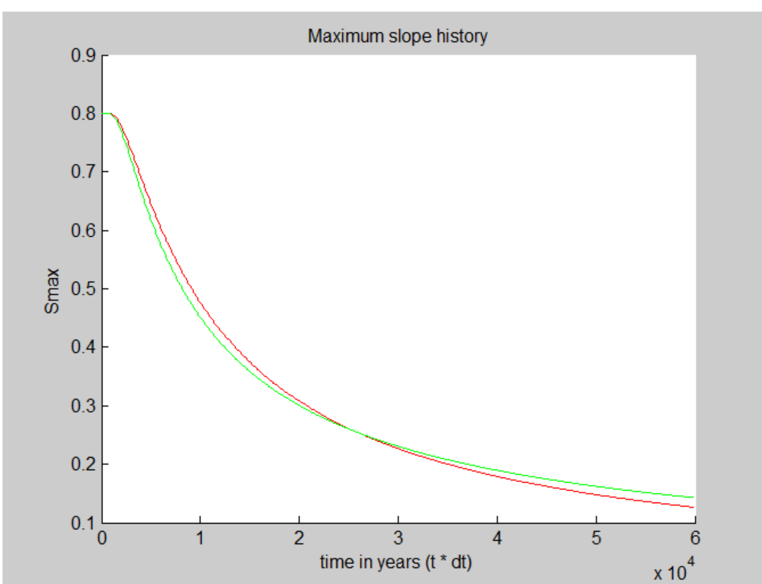


Figure. Output from matlab numerical solution. Red line is the evolution of maximum slope for the triangular ridge case; Green line is the evolution predicted by the modified equation above (tested for a wide range of $h$, $t$, and $K_c$).