I. Flow Mechanics

1. Conservation of Momentum

Objective: derive basic open channel flow equations. Force balance for fluids. Derivation of Boundary Shear Stress (τ_b) and factors that control its magnitude in natural flows.

Definitions:

Linear Momentum = mass - velocity product: mu

Change in momentum = acceleration: $\frac{d}{dt}(mu) = m\frac{du}{dt} = ma$

1A. Kinematics: Isaac Newton (1687)

Newton's Second Law: "change in linear momentum is equal to the sum of forces acting on the body"

$$\sum F = ma$$

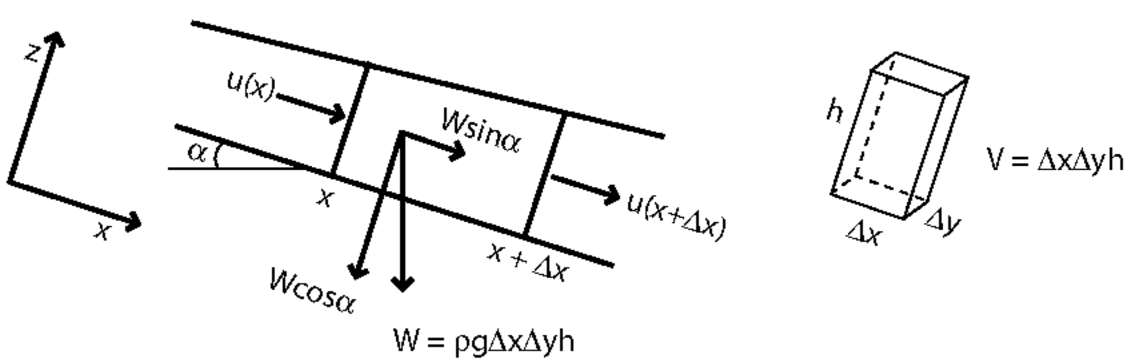
1B. Conservation of Momentum for Fluids

Chauchy's First Law: generalization of Newton's Second Law for a parcel of fluid -- momentum balance for a unit volume of fluid (F_v = force per unit volume)

Consider a volume of fluid (flow depth h and unit bed area (Δx, Δy) of density ρ moving with mean velocity $\bar{u}$:

$$\sum F_v = \rho\frac{d\bar{u}}{dt}$$

We will derive the sum of forces acting on a 1-dimensional flow (velocity only varies in the downslope or x-direction) for simplicity. Variables: flow depth (h), bed surface slope (α) = angle between z and g, and mean velocity ($\bar{u}$), where mean velocity implies the depth-averaged value.



Note that as we will see, water responds to the surface slope, which in the simplest case of uniform depth is equal to the bed slope (α). As bed slope is far easier to measure, it is often used as an approximation of the water surface slope.

Forces acting on a volume of fluid include:

Gravity, Pressure Gradient, Bed Friction

1B.1 Gravity (body force)

Weight = $\rho g \Delta x \Delta y h$ (acts in the vertical direction)
Driving force = downslope component of weight = $\rho g \Delta x \Delta y h \sin\alpha$
(Note: Normal force = $\rho g \Delta x \Delta y h \cos\alpha$)

Driving force per unit volume = $\rho g \Delta x \Delta y h \sin\alpha / (\Delta x \Delta y h) = \rho g \sin\alpha$

1B.2 Pressure gradient (pressure (p) = force per unit area)

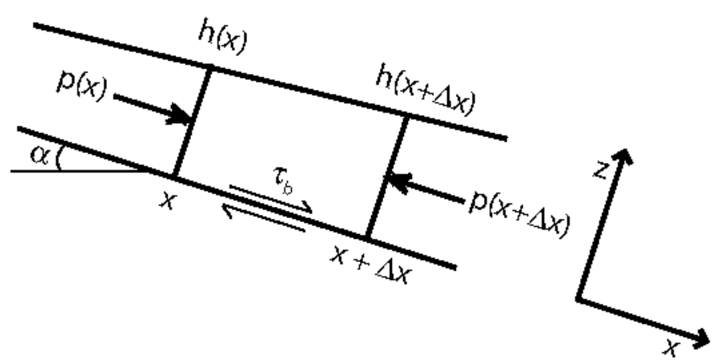
Hydrostatic pressure = weight of the overlying water column per bed area = ρgh. Here we make the small angle approximation that $\cos\alpha \sim 1$, so the fact that h is not measured in the vertical is negligible. This Hydrostatic pressure acts on both the upstream and downstream sides of the unit volume = force per area (Δyh).

There is a net force on the volume only in the presence of a pressure gradient. The pressure gradient is given by the difference between pressure felt on the upstream and the downstream edges of the volume, divided by the width (Δx) of the volume, thus giving a net force per unit volume:

$$\frac{p(x) - p(x + \Delta x)}{\Delta x} = \frac{\rho gh(x) - \rho gh(x + \Delta x)}{\Delta x} = \rho g \frac{\Delta h}{\Delta x} = \rho g \frac{\partial h}{\partial x}$$

1B.3 Bed Friction

Bed friction is described by the Shear Stress (τ_b) acting on the bed. The fluid exerts a shear stress on the bed (oriented downstream), and the bed exerts this same shear stress on the fluid (oriented upstream). Bed friction is the primary source of resistance to flow.



Note that a stress is defined as a force per unit area. Thus, bed friction force per unit volume of fluid (f_v) is given by the basal (or *boundary*) shear stress divided by flow depth (h):

$$f_v = \frac{\tau_b}{h}$$

The condition of an unaccelerated fluid (steady, uniform velocity) of uniform flow depth requires that bed friction must balance the gravitational driving stress:

$$\frac{\tau_b}{h} = \rho g \sin\alpha$$

which is identical for the force balance on a resting block on an inclined plane familiar from freshman physics. A more formal proof of this follows below.

1B.4 Momentum Equation for Fluids (1-dimensional flow)

Chauchy's First Law:
Rate of change of momentum balances sum of forces

$$\rho\frac{d\bar{u}}{dt} = \sum F_v \text{ = gravitational driving force +/- pressure gradient - bed friction}$$

Signs: gravity always drives flow (positive), pressure gradient can either drive flow (depth decreases downstream) or resist flow (depth increases downstream), and bed friction always resists flow (negative).

From 1B.1, 1B.2, 1B.3:

$$\rho\frac{d\bar{u}}{dt} = \rho g \sin\alpha - \rho g \frac{\partial h}{\partial x} - \frac{\tau_b}{h}$$

Note that the sign on the pressure gradient term creates the desired effect.

1C. Temporal and Spatial Momentum Changes

The term $\rho\frac{d\bar{u}}{dt}$ denotes the TOTAL or MATERIAL derivative of momentum, encapsulating BOTH temporal and spatial changes. The total derivative is also called the material derivative because it tracks changes in momentum in a frame of reference that follows a given parcel of water, thus both temporal and spatial variations in flow velocity, and thus momentum are "felt".

How can you isolate temporal and spatial changes?
hint: the mean flow velocity ($\bar{u} = \bar{u}(x,t)$) MUST matter, as the rate at which the moving frame of reference moves downstream essentially determines how a *spatial* change in momentum appears as a *temporal* rate of change.

What does this mean in mathematical terms?
Consider the mean velocity, here constrained to vary only in time and distance downstream

$$\bar{u} = \bar{u}(x,t)$$

Find the derivative of velocity (time and space), using the chain rule:

$$d\bar{u} = \frac{\partial \bar{u}}{\partial t}dt + \frac{\partial \bar{u}}{\partial x}dx$$

The time rate of change in velocity is $\frac{d\bar{u}}{dt}$, so divide through by dt:

$$\frac{d\bar{u}}{dt} = \frac{\partial \bar{u}}{\partial t}\frac{dt}{dt} + \frac{\partial \bar{u}}{\partial x}\frac{dx}{dt}$$

Note that $\frac{dx}{dt} \equiv \bar{u}$, which explains how the material velocity plays a role:

$$\frac{d\bar{u}}{dt} = \frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x}$$

where the first term on the RHS is the rate of change at a fixed point (the *unsteady* term), and the second is the rate of change associated with flow from one point to another (the *convective acceleration* term). Unsteady flow relates to a rising or falling hydrograph, the convective accelerations to flow around bends or over obstructions.

1D. Steady, Uniform Flow

From above we have for conservation of momentum:

$$\rho \frac{d\bar{u}}{dt} = \rho\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x}\right) = \rho g \sin\alpha - \rho g \frac{\partial h}{\partial x} - \frac{\tau_b}{h}$$

Steady flow = no change of velocity in time at a fixed point => $\frac{\partial}{\partial t} = 0$

Uniform flow = no change of velocity in space at a fixed time => $\frac{\partial}{\partial x} = 0$

Under these conditions, the conservation of momentum reduces to: $0 = \rho g \sin\alpha - \rho g \frac{\partial h}{\partial x} - \frac{\tau_b}{h}$

Re-writing we get:

$$\tau_b = \rho g h\left(\sin\alpha - \frac{\partial h}{\partial x}\right)$$

The term in brackets is approximately equal to the water surface slope. Alluvial channels usually have slopes < 5°, and the small angle approximation (sinα ~ tanα = *-dz'/dx'* = S_o; where *z'* is oriented in the vertical and denotes bed elevation) is often used. Thus basal shear stress is often approximated as (using S for water surface slope):

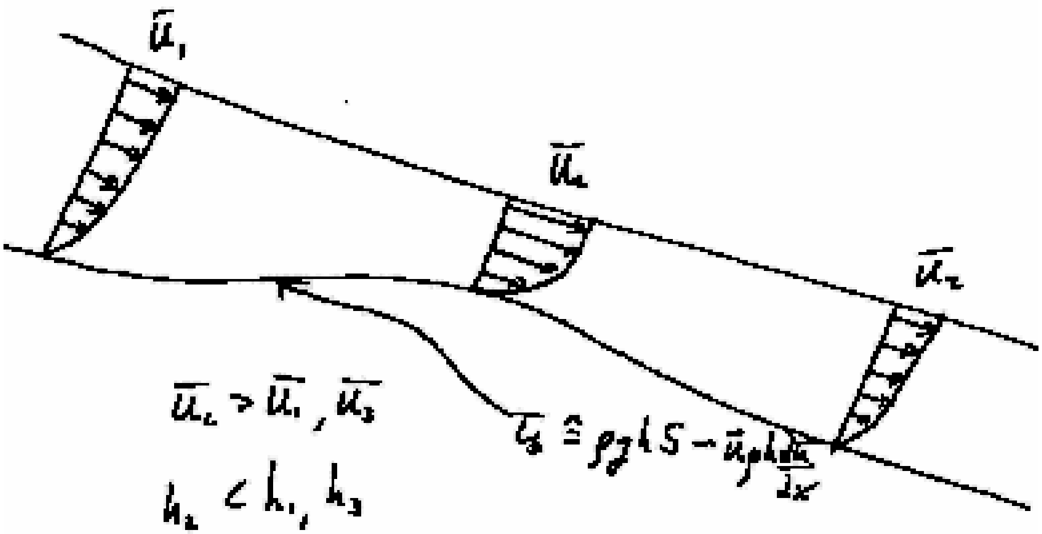
$$\tau_b = \rho g h S$$

(τ_b also related to mean flow velocity and the shape of velocity profiles: next lecture)

Note that this simple expression for the Boundary Shear Stress is in fact a statement of conservation of momentum, albeit under conditions of Steady, Uniform flow. These assumptions are often referred as the condition of "Normal Flow", particularly in the engineering literature.

1E. Steady, Non-Uniform Flow

SKETCH: Flow over a large bar form (1D case), acceleration over rise.



From above we have for conservation of momentum:

$$\rho \frac{d\bar{u}}{dt} = \rho\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x}\right) = \rho g \sin\alpha - \rho g \frac{\partial h}{\partial x} - \frac{\tau_b}{h}$$

Steady flow = no change of velocity in time at a fixed point => $\frac{\partial}{\partial t} = 0$

Under these conditions, the conservation of momentum reduces to:

$$\rho\bar{u}\frac{\partial \bar{u}}{\partial x} = \rho g \sin\alpha - \rho g \frac{\partial h}{\partial x} - \frac{\tau_b}{h}$$

Solving for boundary shear stress under these conditions gives:

$$\tau_b = \rho g h \sin\alpha - \rho g h \frac{\partial h}{\partial x} - \rho \bar{u} h \frac{\partial \bar{u}}{\partial x}$$

$$\tau_b \approx \rho g h S - \rho \bar{u} h \frac{\partial \bar{u}}{\partial x}$$

Thus we see that the acceleration over the obstacle (or around bends) extracts some momentum from the flow such that the boundary shear stress is no longer given simply by the depth-slope product. With field data on the spatial pattern of flow velocities the deviation from "Normal Flow" conditions and the often assumed relation, $\tau_b = \rho g h S$, can be readily evaluated. In most cases the difference is less than a factor of 2.

2. Vertical Velocity Profiles

Objective: derive velocity profiles $u(z)$ as a function of channel slope, depth, roughness

2A. Newtonian Viscous Flow

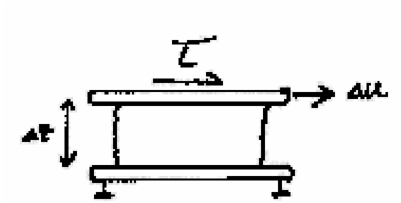
A1. Objective: "constitutive relationship" for water; describes the flow behavior of material.

Relates strain rate to shear stress (ie. response to driving forces acting per unit area on the fluid).

Strain rate (1d): $\frac{\partial u}{\partial z}$; Shear Stress: $\tau(z)$

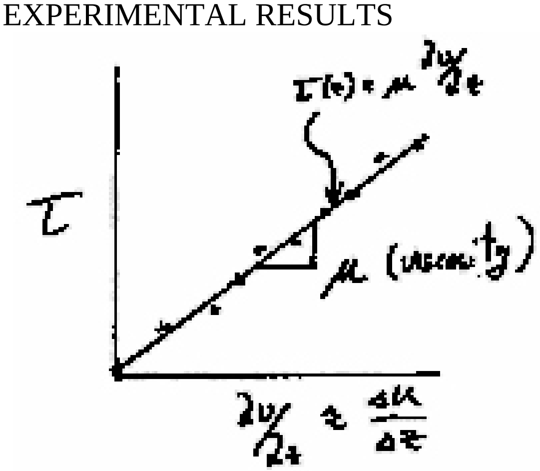
A2. Newton's Experiment

SKETCH OF EXPERIMENTAL SETUP



Measure τ, Δu, Δz; Plot strain rate vs. shear stress; $\frac{\Delta u}{\Delta z} \approx \frac{\partial u}{\partial z}$

SKETCH OF EXPERIMENTAL RESULTS



Regression of experimental data (equation of the line):

$\tau(z) = \mu \frac{\partial u}{\partial z}$ units: [kgm^{-1}s^{-2}] = [kgm^{-1}s^{-1}] [s^{-1}]

units: [Pa] = [Pa s] [s^{-1}]

μ is the viscosity – a material property, function of temperature, results from interaction of molecules (momentum exchange by molecular interaction).

kinematic viscosity: $\nu = \dfrac{\mu}{\rho}$

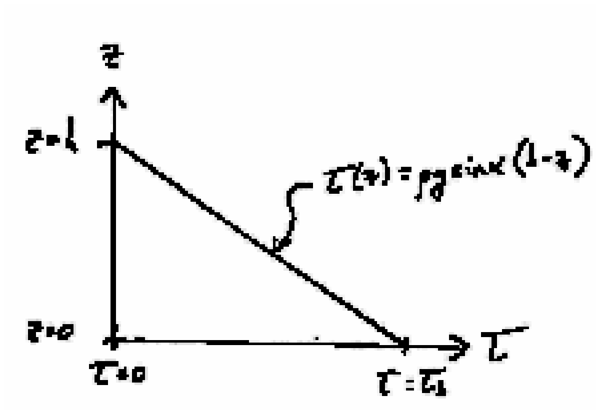
2B. Laminar Flow Velocity Profile

B1. Shear Stress Distribution in an Unaccelerated (Steady Uniform) Flow

As shown earlier, $\tau_b = \rho g h \sin\alpha$ -- for an unaccelerating flow, shear stress on the bed must balance the downslope component of the weight of the overlying fluid (same as for a rigid block on an inclined plane). Recall $\rho g \sin\alpha$ is the driving force per unit volume of fluid due to gravity.

This balance must be true at all levels in the flow, with shear stress reaching a maximum at the bed (deepest water) and decreasing linearly to zero at the surface (neglecting possible wind stress on surface):

$$\tau(z) = \rho g \sin\alpha (h - z)$$



B2. Integration for Velocity Profile

Now that we know the stress – strain rate response of water (laminar conditions) and we know the distribution of stress within the flow, it is possible to combine these to derive the velocity distribution:

$$\frac{\tau(z)}{\mu} = \frac{\partial u}{\partial z} = \frac{\rho g \sin\alpha (h - z)}{\mu}$$

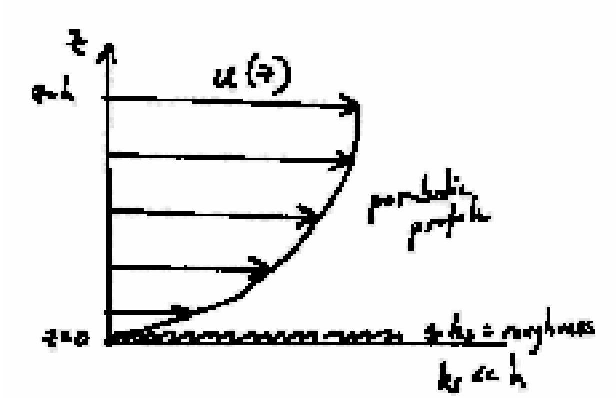
Integrate once with respect to z

$$u(z) = \frac{\rho g \sin\alpha}{\mu}\left(hz - \frac{z^2}{2}\right) + C$$

No slip boundary condition $u(0) = 0$; therefore $C = 0$

Result: Parabolic Velocity profile.
Note: No dependence on boundary roughness in this solution. Why?

SKETCH: velocity profile, define roughness length-scale, k_s



2C. Turbulent Flow

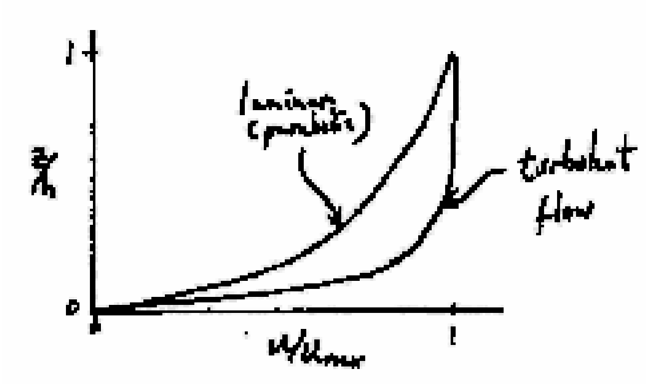
C1. The Problem

Flow of water is unstable – minor perturbations to flow velocity grow rapidly, produce chaotic, highly variable instantaneous velocities (all directions). This chaotic behavior is organized into eddies of a continuum of scales with the largest eddies on the scale of flow depth.

Net effect: much greater resistance to flow – basically the vigorous mixing causes momentum exchange between slow-moving and fast-moving parts of the flow. The effective viscosity (called eddy viscosity) >> μ. Interaction of eddies with the boundary is important and thus roughness becomes an important control on flow velocity.

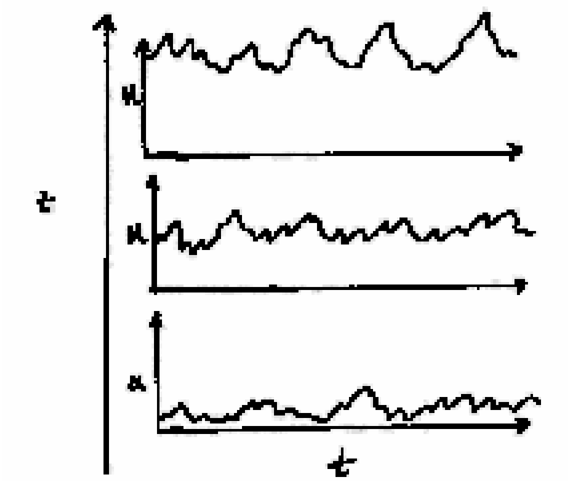
Result: Blunted velocity profile. Shear is concentrated in a narrow band near the bed. As a result, for much of the flow, velocity changes relatively slowly with flow depth (due to effective mixing), such that the mean velocity is a better description of the flow profile than it is in the case of laminar viscous flow (parabolic velocity profile).

SKETCH of laminar and turbulent velocity profiles.



Because of the turbulent fluctuations of instantaneous velocity, the velocity profile represents the time-averaged velocity – averaging out turbulent fluctuations associated with eddies.

SKETCH of 3 panels, $u(t)$ and 3 different levels within the flow.



C2. Condition for the onset of Turbulence.

Reynolds Number (R_e)

Concept: inertial forces (velocity, fluctuations) drive instability
Viscous forces (molecular interactions) dampen instability (smothering the smallest scale eddies)
Where inertial forces dominate → turbulent flow
Where viscous forces dominate → laminar flow

$$R_e \equiv \frac{inertial \;\; forces}{viscous \;\; forces} = \frac{\rho \bar{u} h}{\mu}$$

Note: non-dimensional so equally applicable to all flows. Varying fluid density has same impact as varying viscosity, or flow depth, or velocity, for example.

Empirical studies have defined the critical Reynolds Number for the onset of turbulence.

Pipe flow: $R_{e\_crit} \approx 2000$

Open channel flow: $R_{e\_crit} \approx 500$

C3. Turbulence "closure" and Velocity Profiles

Objective: Develop relation between shear stress, boundary roughness, and turbulent velocity profiles.

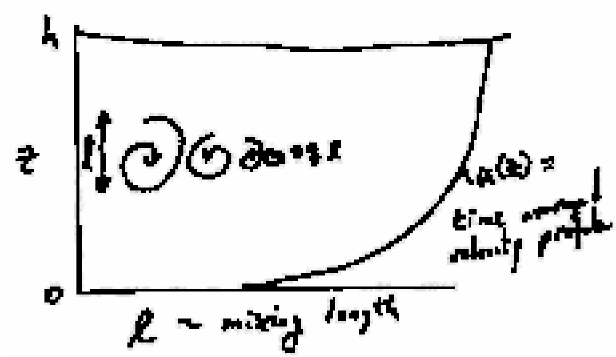
Prandtl Mixing Theory – simple and works well near the bed, good enough for our purposes.

Analogy to laminar flow: Eddy Viscosity

$$\tau(z) = \kappa \frac{\partial u}{\partial z}$$

Critical difference: Eddy viscosity is a function of the flow (velocity, flow depth, slope) *not* a material property like the molecular viscosity μ.
Eddy viscosity describes the exhange of momentum between "layers" of fluid

SKETCH defining *l* the mixing length and cascade of eddies.



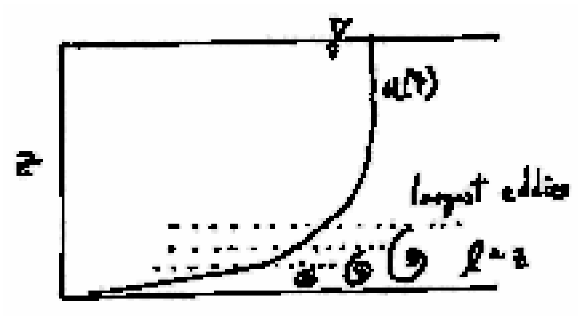
How efficient is the momentum exchange?

$$\Delta momentum \approx \rho \Delta u \approx \rho l \frac{\partial u}{\partial z}$$

Prandtl's intuition (confirmed in experiments): the efficiency of momentum exhange also scales with eddy size, *l*, giving:

$$\kappa \propto \rho l^2 \frac{\partial u}{\partial z}$$

However, near the boundary, eddy size (l) is limited by the distance from the boundary (z)



Thus, near the boundary $l \approx z$, and the eddy viscosity scales as:

$$\kappa \propto \rho z^2 \frac{\partial u}{\partial z}$$

"Near the boundary" technically means $0 < z < 0.2h$.
We also know that near the boundary the turbulent shear stress associated with the eddy viscosity must be approximately equal to the boundary shear stress:

$$\tau_b \approx \kappa \frac{\partial u}{\partial z} \propto \rho z^2 \left(\frac{\partial u}{\partial z}\right)^2$$

To simplify the mathematics, Prandtl introduces the *shear velocity* u_*
By definition:

$$u_* \equiv \sqrt{\frac{\tau_b}{\rho}} \qquad \text{units } [\mathrm{ms^{-1}}] \text{ velocity units}$$

Thus Prandtl mixing theory predicts the following relation for shear velocity:

$$u_* \propto \sqrt{\rho z^2 \left(\frac{\partial u}{\partial z}\right)^2 \Big/ \rho} \propto z \frac{\partial u}{\partial z}$$

From experiments this is verified to have a constant proportionality coefficient (called Von Karman's constant, k)

$$u_* = kz \frac{\partial u}{\partial z} \quad ; \text{ where } k = 0.4$$

Dimensional analysis confirms that k is a constant (unitless).

C4. The Law of the Wall

Prandtl mixing theory gives us a simple parameterization of turbulent flow which can be used to determine the velocity profile:

$$\frac{\partial u}{\partial z} = \frac{u_*}{k}\frac{1}{z}$$

Integrate once with respect to z

$$u(z) = \frac{u_*}{k}\ln(z) + C$$

Boundary needed condition to find C: $u(z)$ goes to zero at an elevation (just above the bed), z_o. In fact turbulent flow does *not* reach the boundary – a viscous sub-layer is always present, so the turbulent flow law can not be extended to the boundary. z_o is an effective roughness parameter, related to grainsize, bedform size, and other effects. It is not a physically meaningful height (velocity really goes to zero on the boundary – z_o dodges the problem of needing to "match" the turbulent velocity profile to the very thin viscous profile in the sublayer.)

So, at $z = z_o$, $u(z) = 0$:

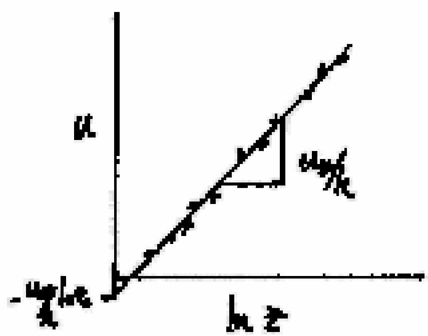
$$u(z_o) = 0 = \frac{u_*}{k}\ln(z_o) + C$$

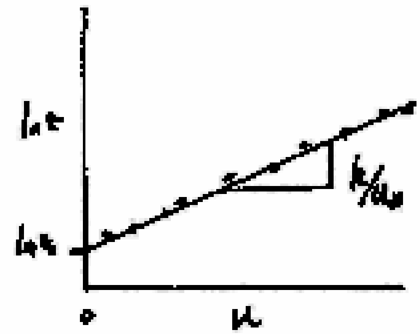
$$C = -\frac{u_*}{k}\ln(z_o)$$

$$u(z) = \frac{u_*}{k}\ln\left(\frac{z}{z_o}\right)$$

Final expression is the so-called "Law of the Wall". Technically it is only valid near the boundary ($0 < z < 0.2h$) where Prandtl mixing theory applies, but is often extended with reasonable accuracy to the full depth profile – it will be good enough for our purposes. [Problems of suspended sediment transport, not considered in detail in this class, will be more sensitive to details of the vertical velocity structure].

SKETCH logarithmic velocity profile, discuss determination of z_o and shear stress (τ_b) from velocity profile data.





Regression Analysis:
ln(z) on y-axis, *u*(z) on x-axis; equation of the line:

$\ln z = \frac{k}{u_*} u(z) + \ln z_o$ … slope of line gives k/u_*, intercept $\ln(z_o)$

u(*z*) on y-axis, ln(*z*) on x-axis; equation of the line:

$u(z) = \frac{u_*}{k} \ln z - \frac{u_*}{k} \ln z_o$ .. slope of line gives u_*/k, intercept $-(u_*/k)\ln(z_o)$

$$\tau_b = \rho u_*^{\ 2}$$

In next lecture we will discuss the factors that influence the value of z_o, the roughness parameter, including the thickness of the viscous sub-layer relative to the boundary roughness. We will consider two endmember states: hydraulically smooth flow (HSF) and hydraulically rough flow (HRF).

C5. Vertically Averaged Velocity & the 4/10ths Rule

To find the vertical average velocity, integrate once (form the bed $z = 0$ to the surface $z = h$) and divide by flow depth (h)

$$\langle u \rangle \equiv \frac{1}{h}\int_0^h u(z)dz = \frac{1}{h}\int_0^h \frac{u_*}{k}\ln\left(\frac{z}{z_o}\right)dz$$

Recall $\int \ln ax = x \ln ax - x$

$$\langle u \rangle = \frac{1}{h}\left[\frac{u_*}{k} z\left(\ln\left(\frac{z}{z_o}\right) - 1\right)\Big|_{z=h}\right]$$

$$\langle u \rangle = \frac{u_*}{k}\left(\ln\left(\frac{h}{z_o}\right) - 1\right)$$

This can also be written as:

$$\langle u \rangle = \frac{u_*}{k}\left(\ln\left(\frac{h}{z_o}\right) + \ln(0.37)\right)$$

$$\langle u \rangle = \frac{u_*}{k}\ln\left(\frac{0.37h}{z_o}\right) = u(z)\big|_{z=0.37h} = u(0.37h)$$

This is called the 4/10ths rule: the velocity measured at ~4/10 flow depth up from the bed should be approximately equal to the mean velocity.

Another rule of thumb is that mean velocity is about 8/10s of the surface velocity (which is easily measured).

Next lecture: Hydraulic formulae for open channel flow -- Alternative engineering approximations for cross-sectionally averaged velocity in channels. These involve various formulations of how to represent boundary roughness and how to estimate roughness parameters from field data.

3. Hydraulic Formulae for Open Channel Flow

Objective: review relations for cross-sectional mean velocity as a function of channel slope, depth, roughness, and methods for measuring the roughness parameters.

FACTORS INFLUENCING HYDRAULIC ROUGHNESS

Bed material size (D_{50}, D_{84}, k_s, z_o, n_g); Relative roughness (h/D_{50}); Presence of sediment transport (momentum extraction); Bedforms and barforms; Vegetation; Obstructions (tree stumps, logs, boulders, bedrock outcrops, ect); Variations in channel width and depth; Channel curvature (sinuosity)

METHODS FOR ESTIMATING ROUGHNESS PARAMETERS

"Roughness" is represented in various ways in familiar flow velocity equations. We will consider: Chezy's equation, Manning's equation, the Darcy-Weisbach equation, and a generalized D-W equation (all for average velocity), and the "Law of the Wall" equation for the velocity profile or a turbulent flow near a boundary (logarithmic).

Variables Used:

S	: Water surface slope (= bed slope for steady uniform flow) [m/m]
R_h	: Hydraulic radius (R_h = A/P = flow depth for infinitely wide channel) [m]
A	: Cross-sectional area [m^2]
P	: Wetted perimeter [m]
Q	: Water Discharge [m^3/s]
$\overline{U}$	: Cross-sectionally averaged velocity [m/s]
z	: cartesian coordinate (perpendicular to bed) [m]
h	: flow depth (perpendicular to bed) [m]
τ_b	: basal shear stress [Pa]
k	: von Karman's constant = 0.40
C	: Chezy roughness coefficient [$m^{1/2}$/s]
f	: Darcy-Weisbach friction factor [ ]
n	: Manning's roughness factor [s/$m^{1/3}$]
C_f	: Generalized non-dimensional friction factor [ ]
k_s	: grain roughness scale ~ D_{84}

Chezy's Equation:

$$\frac{Q}{A} = \overline{u} = C\sqrt{R_h S}$$

what are the units of C? [$\sqrt{g}$]

Manning's Equation: (metric units!!)
(1840's; observed chezy's C = function of depth)

$$\frac{Q}{A} = \bar{u} = \frac{1}{n} R_h^{2/3} S^{1/2}$$

what are the units of n?

Darcy-Weisbach Equation: (pipe flow & theory; f is non-dimensional)

$$\bar{u}^2 = \frac{8gR_hS}{f}$$

Generalized Darcy-Weisbach: (the 8 coefficient above is only for pipes)

$$\bar{u} = \frac{\sqrt{gR_hS}}{C_f^{1/2}} \quad ; \quad \tau_b = \rho C_f \bar{u}^2 \quad \text{(for } R_h \sim h\text{)}$$

Law of the Wall:

(for turbulent flow, applies strictly just near the boundary, z < .2h, but works fairly well for entire profile)

$$u = \frac{u_*}{k} \ln \frac{z}{z_o}$$

where u* = $\sqrt{\frac{\tau_b}{\rho}}$, "shear velocity"
k = 0.40 (Von Karman's Constant)

z_o is the point where idealized velocity profile goes to zero (a fictional level in the flow)

Integrating over flow depth and dividing by h (for vertically averaged velocity):

$$\langle u \rangle = \frac{u_*}{k}\left(\ln\frac{h}{z_o} - 1\right)$$

The 4/10s Rule:

$$\langle u \rangle = \frac{u_*}{k}\left(\ln\frac{h}{z_o} + \ln(.37)\right) = \frac{u_*}{k}\ln\frac{.37h}{z_o} = u(z = .37h)$$

I. Visual Estimates of Manning's n:

1. Visual estimate of field conditions using experience, "type" photographs, and published tables. Tables are found in most geomorphology texts. "Type" photos are in Water Supply Paper 1849. Listed below are a few examples (from Richards):

Description	Manning's n
Artificial channel, concrete	.014
Excavated channel, earth	.022
Excavated channel, gravel	.025
Natural channel, < 30 m wide, clean, regular	.030
Natural channel, < 30 m wide, some weeds, stones	.035
Mountain stream, cobbles, boulders	.050
Major stream, > 30 m wide, clean, regular	.025

2. Estimate from Table given by Chow (1959), where n is given by:

$$n = (n_0 + n_1 + n_2 + n_3 + n_4)\, m_5$$

Material, n_0		Degree of Irregularity, n_1		Variation of cross-section, n_2	
earth	.020	smooth	.000	gradual	.000
rock	.025	minor	.005	alt. occasionally	.005
fine gravel	.024	moderate	.010	alt. frequently	.010-.015
coarse grav.	.028	severe	.020		

Channel obstructions, n_3		Vegetation n_4		Degree of meandering, m_5	
negligible	.000	low	.005-.010	none	1.000
minor	.010-.015	medium	.010-.025	minor	1.000
appreciable	.020-.030	high	.025-.050	appreciable	1.150
severe	.040-.060	v.high	.050-.100	severe	1.300

II. Empirical relationship between the Darcy-Weisbach friction factor and grainsize and flow depth (Leopold et al., 1964).

Empirical data fits the line:

$$\frac{1}{\sqrt{f}} = 2.0\log\left(\frac{h}{D_{84}}\right) + 1.0$$

see figure, next page.

D_{84} = 84th percentile value from cum. freq. distribution (grain diameter)

III. Back-calculation of **n** or **f** from field data using velocity equations given above.

$$\overline{u} = \frac{1}{n} R_h^{2/3} S^{1/2}$$

S = slope of the water surface

Method: u (cross-sectional average), R, and S are measured,
n and/or f is back-calculated.

IV. Calcualtion of local hydrodynamic roughness ("grain roughness": z_o) from velocity profiles using the Law of the Wall.

$$u = \frac{u_*}{k} \ln \frac{z}{z_o}$$

where $u_* = \sqrt{\frac{\tau_b}{\rho}}$, k = 0.40 (Von Karman's Constant)

First we must define hydraulically rough (HRF) vs. hydraulically smooth (HSF) flow. Given that k_s = grain diameter, $\delta\nu$ = thickness of the viscous sub-layer, and ν = kinematic viscosity, we define the shear Reynolds number (R_*) as

$$R_* = \frac{u_* k_s}{\nu}$$

HSF occurs where $R_* < 3$, and HRF where $R_* > 100$, from Nikaradse's data.

Case 1. HSF:

$$z_o = \frac{\nu}{9u_*}$$

Case 2. HRF:

$$z_o = \frac{k_s}{30} \; ; \qquad k_s \sim D_{84} \text{ (grain roughness)}$$

If $3 < R_* < 100$, then find z_o form Nikaradse's diagram, see next page.

Note, for typical river temperatures, $\nu = 1.514 \times 10^{-2}$ cm2/s.