Surface Processes and Landforms (12.163/12.463)
Fall 02 -- K. Whipple

FACTORS INFLUENCING HYDRAULIC ROUGHNESS

Bed material size (D_{50}, D_{84}, k_s, z_o, n_g); Relative roughness (h/D_{50}); Presence of sediment transport (momentum extraction); Bedforms and barforms; Vegetation; Obstructions (tree stumps, logs, boulders, bedrock outcrops, ect); Variations in channel width and depth; Channel curvature (sinuosity)

METHODS FOR ESTIMATING ROUGHNESS PARAMETERS

"Roughness" is represented in various ways in familiar flow velocity equations. We will consider: Chezy's equation, Manning's equation, the Darcy-Weisbach equation, and a generalized D-W equation (all for average velocity), and the "Law of the Wall" equation for the velocity profile or a turbulent flow near a boundary (logarithmic).

Variables Used:

S : Water surface slope (= bed slope for steady uniform flow) [m/m]
R_h : Hydraulic radius ($R_h = A/P$ = flow depth for infinitely wide channel) [m]
A : Cross-sectional area [m^2]
P : Wetted perimeter [m]
Q : Water Discharge [m^3/s]
$\overline{u}$: Cross-sectionally averaged velocity [m/s]
z : cartesian coordinate (perpendicular to bed) [m]
h : flow depth (perpendicular to bed) [m]
τ_b : basal shear stress [Pa]
k : von Karman's constant = 0.40
C : Chezy roughness coefficient [$m^{1/2}/s$]
f : Darcy-Weisbach friction factor [ ]
n : Manning's roughness factor [$s/m^{1/3}$]
C_f : Generalized non-dimensional friction factor [ ]
k_s : grain roughness scale ~ D_{84}

Chezy's Equation:

$$\frac{Q}{A} = \overline{u} = C\sqrt{R_h S}$$

without looking at the variable list above, work out the units of C.

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Manning's Equation: (metric units!!)
(1840's; observed chezy's C = function of depth)

$$\frac{Q}{A} = \bar{u} = \frac{1}{n} R_h^{2/3} S^{1/2}$$

what are the units of n?

Darcy-Weisbach Equation: (pipe flow & theory; f is non-dimensional)

$$\bar{u}^2 = \frac{8gR_hS}{f}$$

Generalized Darcy_Weisbach:

$$\bar{u} = \frac{\sqrt{gR_hS}}{C_f^{1/2}} \quad ; \quad \tau_b = \rho C_f \bar{u}^2 \quad \text{(for } R_h \sim h\text{)}$$

Law of the Wall:

(for turbulent flow, applies strictly just near the boundary, z < .2h, but works fairly well for entire profile)

$$u = \frac{u_*}{k} \ln \frac{z}{z_o}$$

where $u^* = \sqrt{\frac{\tau_b}{\rho}}$, "shear velocity"
k = 0.40 (Von Karman's Constant)

z_o is the point where idealized velocity profile goes to zero (a fictional level in the flow)

Integrating over flow depth and dividing by h (for vertically averaged velocity):

$$\langle u \rangle = \frac{u_*}{k}\left(\ln \frac{h}{z_o} - 1 \right)$$

The 4/10s Rule:

$$\langle u \rangle = \frac{u_*}{k}\left(\ln \frac{h}{z_o} + \ln(.37) \right) = \frac{u_*}{k} \ln \frac{.37h}{z_o} = u(z = .37h)$$

I. Visual Estimates of Manning's n:

1. Visual estimate of field conditions using experience, "type" photographs, and published tables. Tables are found in most geomorphology texts. "Type" photos are in Water Supply Paper 1849. Listed below are examples (from Richards):

Description	Manning's n
Artificial channel, concrete	.014
Excavated channel, earth	.022
Excavated channel, gravel	.025
Natural channel, < 30 m wide, clean, regular	.030
Natural channel, < 30 m wide, some weeds, stones	.035
Mountain stream, cobbles, boulders	.050
Major stream, > 30 m wide, clean, regular	.025

2. Estimate from Table given by Chow (1959), where n is given by:

$$n = (n_0 + n_1 + n_2 + n_3 + n_4)\, m_5$$

Material, n_0	
earth	.020
rock	.025
fine gravel	.024
coarse grav.	.028

Degree of Irregularity, n_1	
smooth	.000
minor	.005
moderate	.010
severe	.020

Variation of cross-section, n_2	
gradual	.000
alt. occasionally	.005
alt. frequently	.010-.015

Channel obstructions, n_3	
negligible	.000
minor	.010-.015
appreciable	.020-.030
severe	.040-.060

Vegetation n_4	
low	.005-.010
medium	.010-.025
high	.025-.050
v.high	.050-.100

Degree of meandering, m_5	
none	1.000
minor	1.000
appreciable	1.150
severe	1.300

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II. Empirical relationship between the Darcy-Weisbach friction factor and grainsize and flow depth (Leopold et al., 1964).

Empirical data fits the line:

$$\frac{1}{\sqrt{f}} = 2.0\log\left(\frac{h}{D_{84}}\right) + 1.0$$

see figure, next page.

D_{84} = 84th percentile value from cum. freq. distribution (grain diameter)

III. Back-calculation of n or f from field data using velocity equations given above.

$$\bar{u} = \frac{1}{n} R_h^{2/3} S^{1/2}$$

S = slope of the water surface

Method: u (cross-sectional average), R, and S are measured, n and/or f is back-calculated.

IV. Calcualtion of local hydrodynamic roughness ("grain roughness": z_o) from velocity profiles using the Law of the Wall.

$$u = \frac{u_*}{k} \ln \frac{z}{z_o}$$

where $u_* = \sqrt{\frac{\tau_b}{\rho}}$, $k = 0.40$ (Von Karman's Constant)

First we must define hydraulically rough (HRF) vs. hydraulically smooth (HSF) flow. Given that k_s = grain diameter, $\delta\nu$ = thickness of the viscous sub-layer, and ν = kinematic viscosity, we define the shear Reynolds number (R_*) as

$$R_* = \frac{u_* k_s}{\nu}$$

HSF occurs where $R_* < 3$, and HRF where $R_* > 100$, from Nikaradse's data.

Case 1. HSF:

$$z_o = \frac{\nu}{9u_*}$$

Case 2. HRF:

$$z_o = \frac{k_s}{30} \quad ; \qquad k_s \sim D_{84} \text{ (grain roughness)}$$

If $3 < R_* < 100$, then find z_o form Nikaradse's diagram, see next page.
Note, for typical river temperatures, $\nu = 1.514 \times 10^{-2}$ cm2/s.

Table 6.1 **Manning roughness coefficients (n) for different boundary types.**

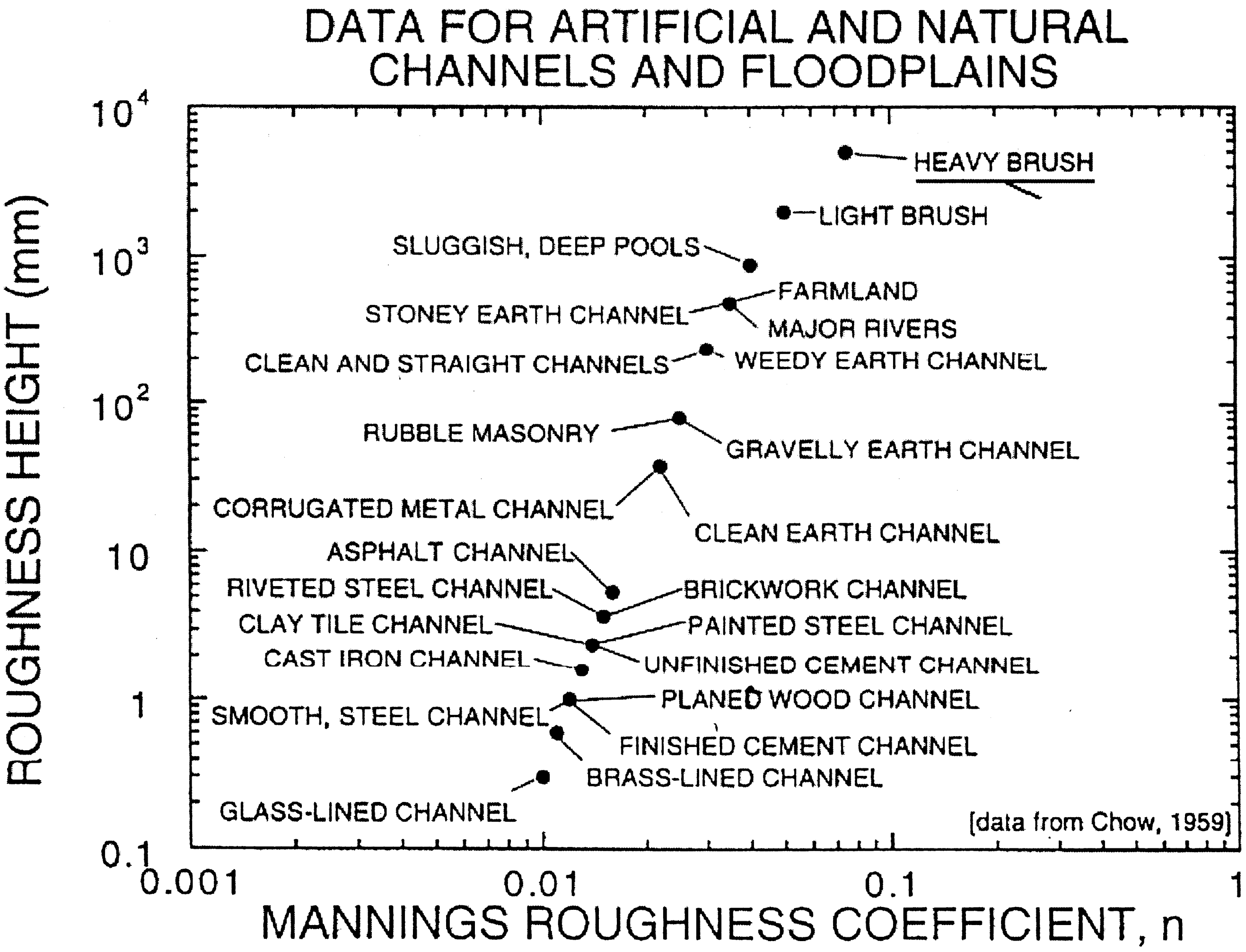
Boundary	Manning roughness $n, ft^{1/6}$
Very smooth surfaces such as glass, plastic, or brass	0.010
Very smooth concrete and planed timber	0.011
Smooth concrete	0.012
Ordinary concrete lining	0.013
Good wood	0.014
Vitrified clay	0.015
Shot concrete, untroweled, and earth channels in best condition	0.017
Straight unlined earth canals in good condition	0.020
Rivers and earth canals in fair condition—some growth	0.025
Winding natural streams and canals in poor condition—considerable moss growth	0.035
Mountain streams with rocky beds and rivers with variable sections and some vegetation along banks	0.040–0.050

From Albertson, M. L. and Simons, D. B. 1964, Fluid Mechanics. In *Handbook of Applied Hydrology* edited by Ven T. Chow. Copyright © 1964 by McGraw-Hill Inc. Used with permission of McGraw-Hill Book Company.

Table 8.1 Values of Manning's roughness coefficient for various types of natural channel

CHANNEL TYPE	NORMAL VALUE	RANGE
Small channels (*width <30 m*)		
Low-gradient streams		
Unvegetated straight channels at bankfull stage	0.030	0.025–0.033
Unvegetated winding channels with some pools and shallows	0.040	0.033–0.045
Winding vegetated channels with stones on bed	0.050	0.045–0.060
Sluggish vegetated channels with deep pools	0.070	0.050–0.080
Heavily vegetated channels with deep pools	0.100	0.075–0.150
Mountain streams (with steep unvegetated banks)		
Few boulders on channel bed	0.040	0.030–0.050
Abundant cobbles and large boulders on channel bed	0.050	0.040–0.070
Large channels (*width >30 m*)		
Regular channel lacking boulders or vegetation	—	0.025–0.060
Irregular channel	—	0.035–0.100

Source: Based on data in V. T. Chow (ed.) (1964) *Handbook of Applied Hydrology*. McGraw-Hill, New York.

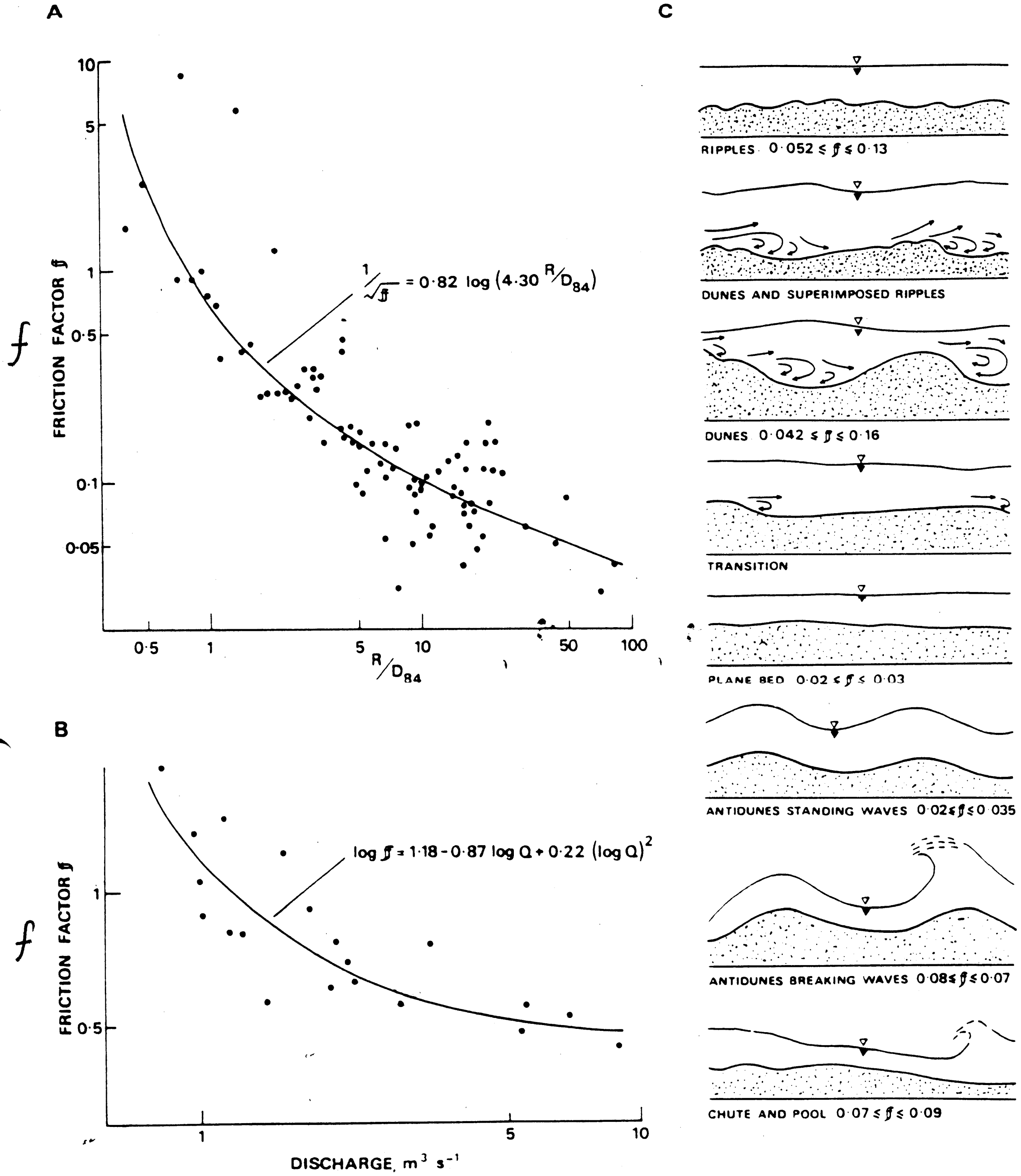


[data from Chow, 1959]

Table 3.2 Average resistance coefficients for straight channels in various conditions.

Type of channel and description	n	f_f†	C†
Artificial channel, shuttered concrete	0.014	0.016	71
Excavated channel, earth	0.022	0.039	45
Excavated channel, gravel	0.025	0.049	40
Natural channel <30 m wide, clean, regular	0.030	0.072	33
Natural channel <30 m wide, some weeds, stones	0.035	0.093	29
Natural channel <30 m wide, sluggish weedy pools	0.070	0.400	14
Mountain streams, cobbles and boulders	0.050	0.196	20
Major streams >30 m, clean, regular	0.025	0.049	40

†for hydraulic radius, R, of 1 m.



D-W

Fig 3.4 A. Relationship between friction factor and relative roughness (data of Leopold and Wolman, 1957; Limerinos, 1970; Charlton et al., 1978; Hey, 1979).
B. Relationship between friction factor and discharge at a sandy-bed cross-section, River Bollin.
C. The sequence of bed forms related to increasing flow intensity, with corresponding values of the Darcy-Weisbach friction factor in flume experiments (after Simons and Richardson, 1966).

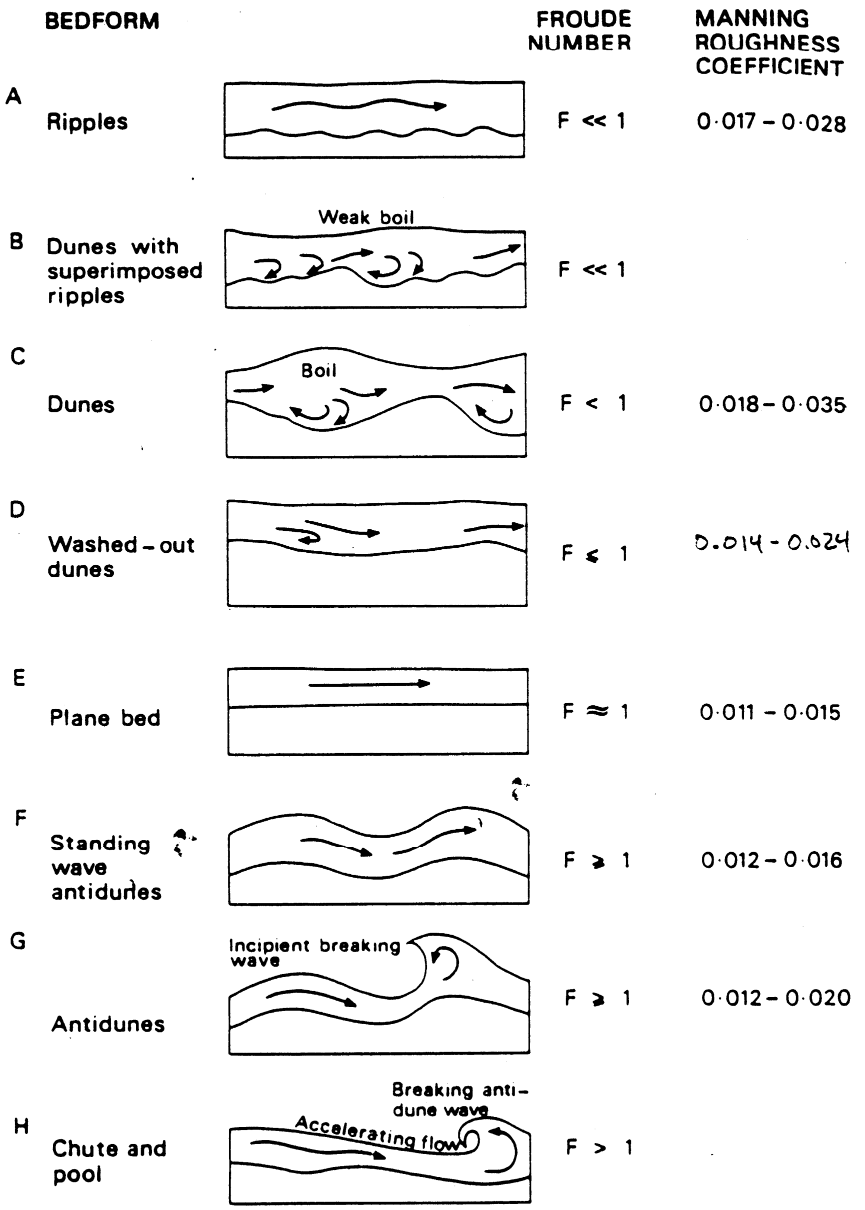


Fig. 8.10 *Bedforms in sandy alluvial channels in relation to flow regimes expressed by Froude numbers. At low flow velocities ripples are formed (A), but as the flow velocity increases ripples are transformed into larger forms called* ***dunes*** *(B and C), both being out of phase with waves on the water surface. With a further increase in velocity bed undulations are planed off, resistance to flow is lowered and sediment transport rates increase (D and E). This is a transitional state between subcritical and supercritical flow. With a further increase in velocity, supercritical flow gives rise to* ***antidunes*** *which because they are in phase with standing waves at the water surface present a low resistance to flow (F and G). Antidunes move upstream since sediment is lost from their downstream side more rapidly than it is deposited. At the highest flow velocities fast – flowing shallow chutes alternate with deeper pools (H). (Based on D. B. Simons and E. V. Richardson (1963)* Transactions of the American Society of Civil Engineers, ***128****, Fig. 2, p. 289.)*

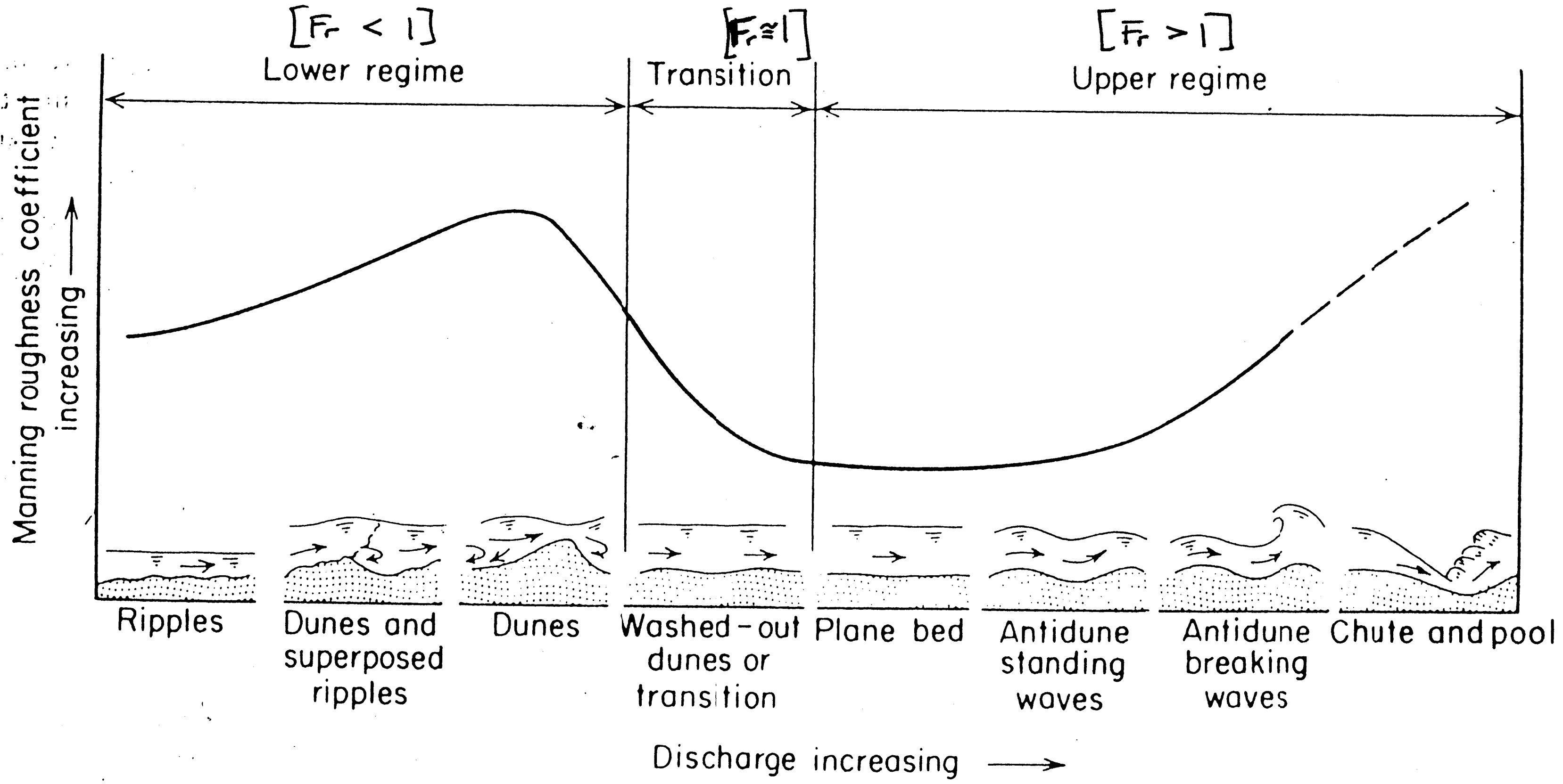
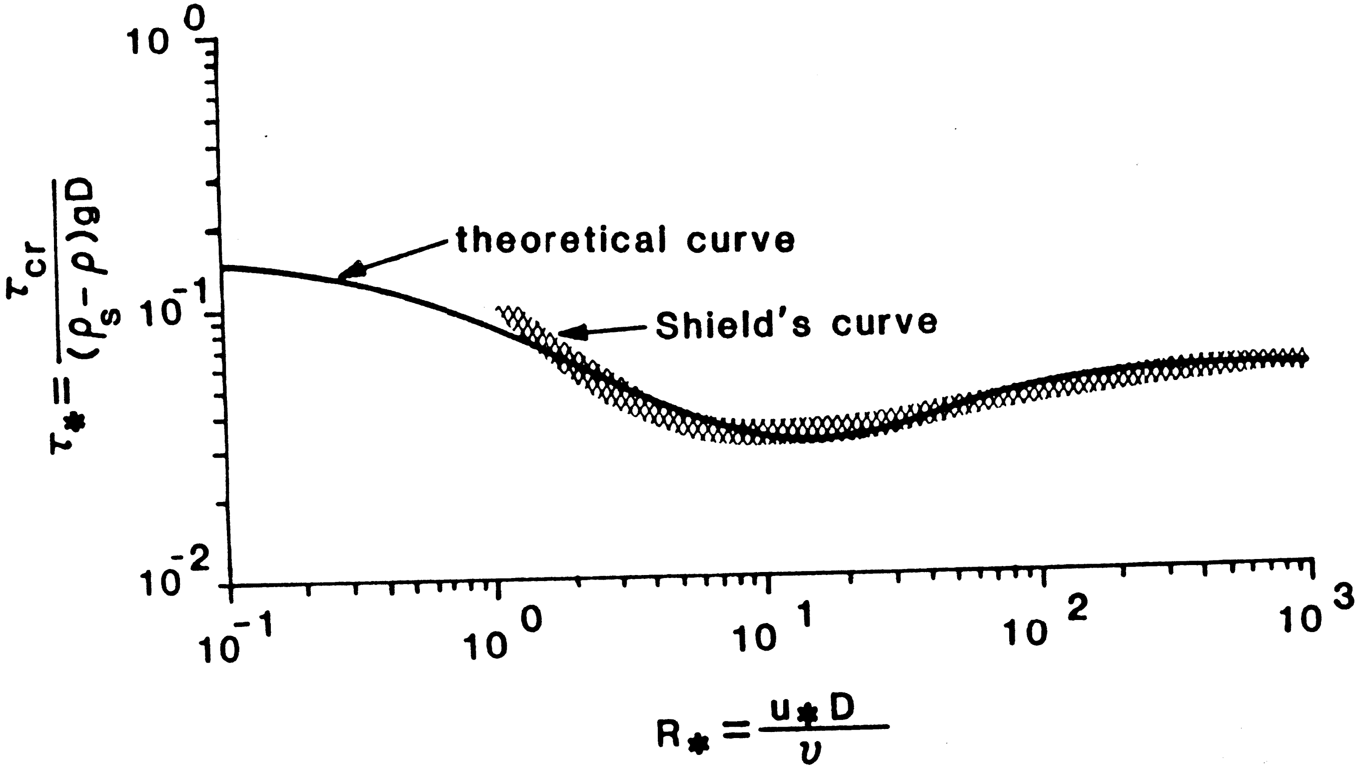
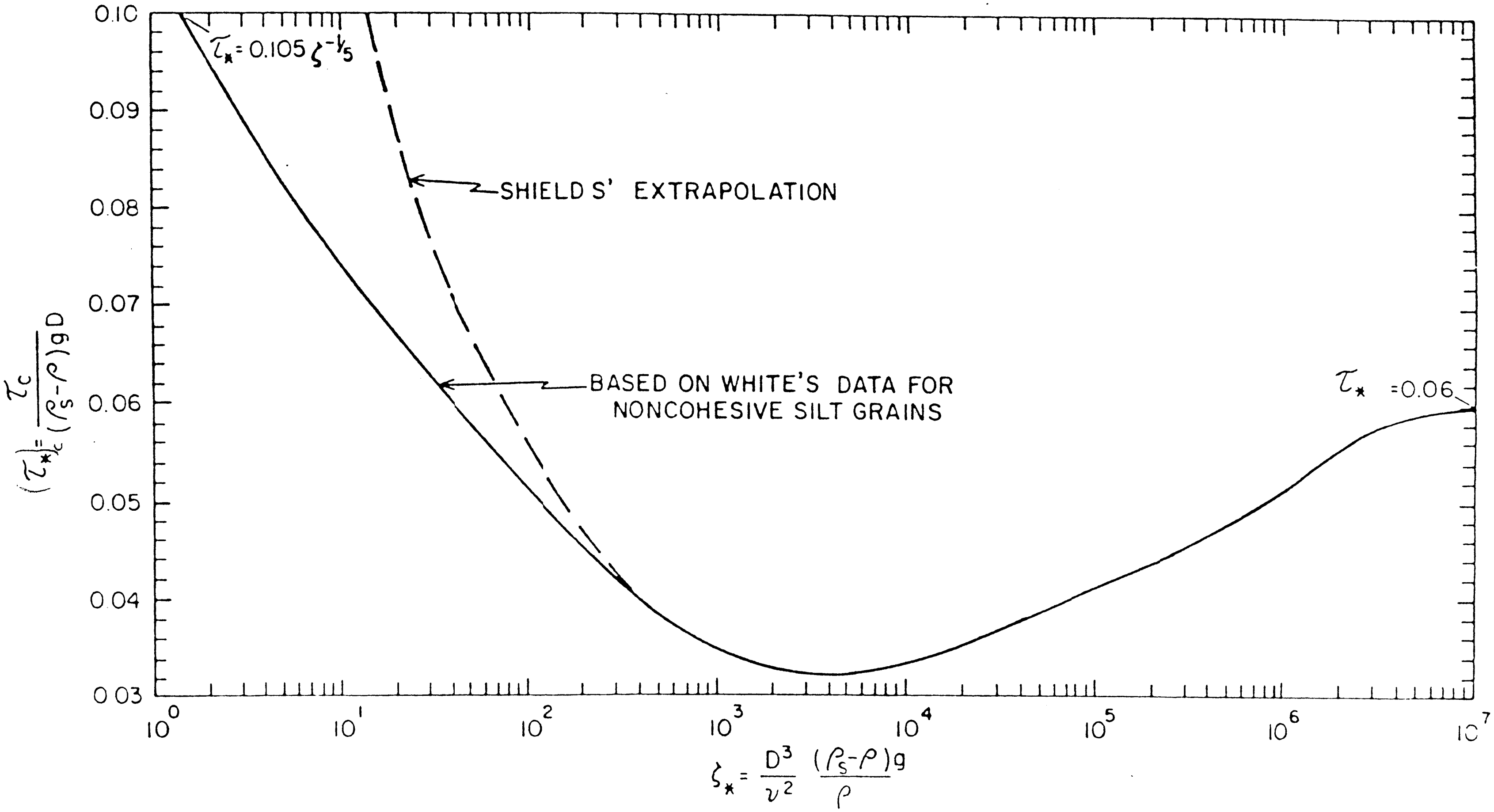
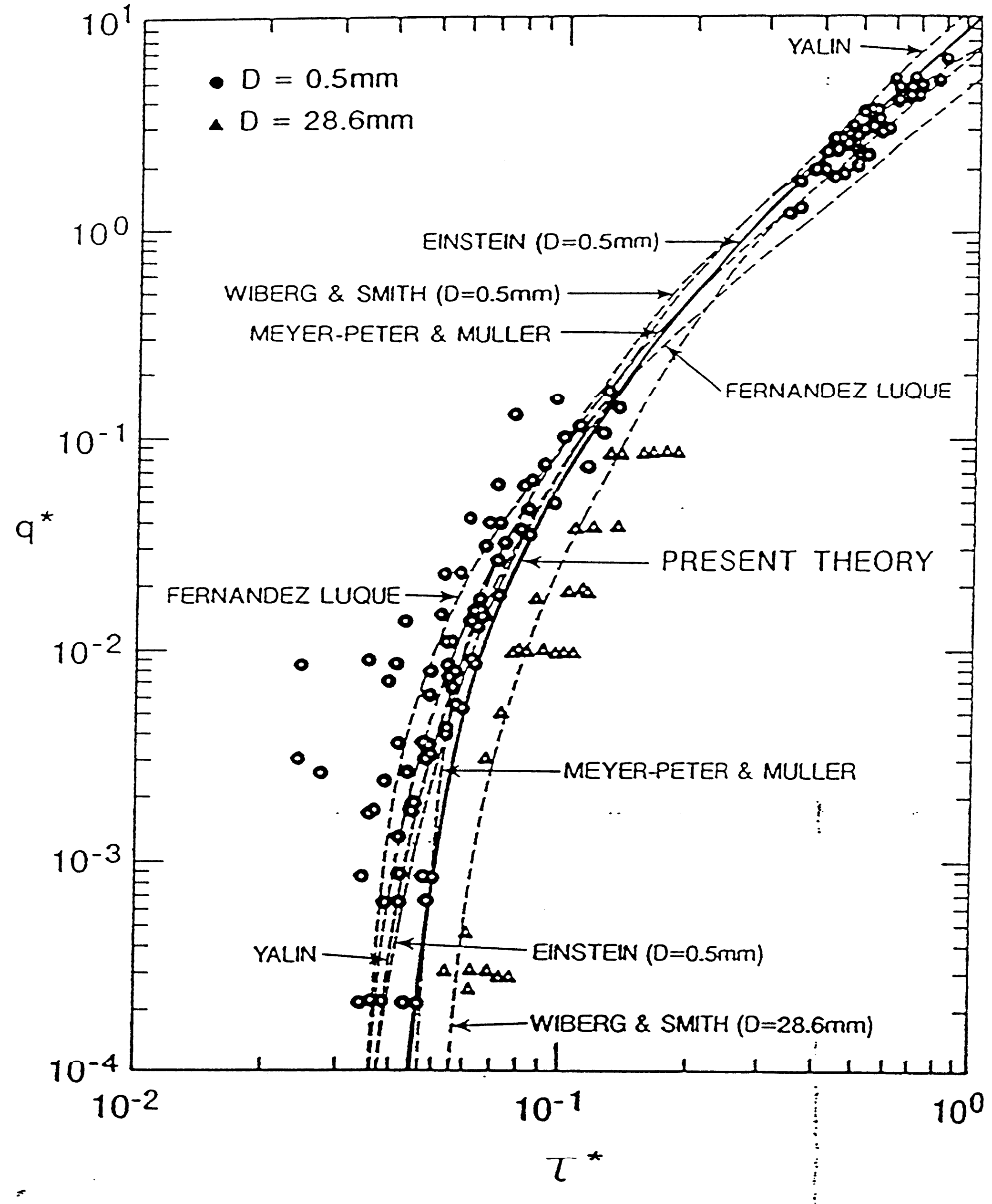


Figure 12.6 Variations in bedforms and bed roughness as transport of a uniform sandy material increases with discharge. The upper part of the transition regime is associated with a Froude number of unity.
Source: Coastal Eng. Div., Am. Soc. Civil Eng., vol. 99, pp. 231–43, after Simons *et al.*


$\tau_* = \frac{\tau_{cr}}{(\rho_s - \rho)gD}$
10^0
10^{-1}
10^{-2}
theoretical curve
Shield's curve
10^{-1}
10^0
10^1
10^2
10^3
$R_* = \frac{u_* D}{\upsilon}$


0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
$(\tau_*)_c = \frac{\tau_c}{(\rho_s-\rho)gD}$
$\tau_* = 0.105\,\zeta^{-1/5}$
SHIELD S' EXTRAPOLATION
BASED ON WHITE'S DATA FOR NONCOHESIVE SILT GRAINS
$\tau_* = 0.06$
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
$\zeta_* = \frac{D^3}{\nu^2}\,\frac{(\rho_s-\rho)g}{\rho}$



Wiberg + Smith, 1989

FIG. 4

VISCOSITY AND FLUIDITY OF WATER
0-100° C

Bingham and Jackson, Bull. Bur. Stds. 11, 75 (1918)

Temp. °C	Fluidity 1/poises	Viscosity centi-poises	Specific viscosity (0° C)	Temp. °C	Fluidity 1/poises	Viscosity centi-poises	Specific viscosity (0° C)
0	55.80	1.7921	1.0000	50	182.00	0.5494	0.3066
1	57.76	1.7313	.9661	51	185.05	.5404	.3015
2	59.78	1.6728	.9334	52	188.14	.5315	.2966
3	61.76	1.6191	.9035	53	191.23	.5229	.2918
4	63.80	1.5674	.8746	54	194.34	.5146	.2871
5	65.84	1.5188	.8475	55	197.45	.5064	.2826
6	67.90	1.4728	.8218	56	200.62	.4985	.2782
7	70.01	1.4284	.7971	57	203.78	.4907	.2738
8	72.15	1.3860	.7734	58	206.95	.4832	.2696
9	74.28	1.3462	.7512	59	210.13	.4759	.2656
10	76.47	1.3077	.7297	60	213.33	.4688	.2616
11	78.66	1.2713	.7094	61	216.54	.4618	.2577
12	80.89	1.2363	.6899	62	219.80	.4550	.2539
13	83.14	1.2028	.6712	63	223.07	.4483	.2502
14	85.40	1.1709	.6534	64	226.34	.4418	.2465
15	87.69	1.1404	.6363	65	229.61	.4355	.2430
16	90.00	1.1111	.6200	66	232.94	.4293	.2396
17	92.35	1.0828	.6042	67	236.25	.4233	.2362
18	94.71	1.0559	.5892	68	239.57	.4174	.2329
19	97.10	1.0299	.5747	69	242.91	.4117	.2297
20*	99.50	1.0050	.5608	70	246.26	.4061	.2266
21	101.94	.9810	.5474	71	249.63	.4006	.2235
22	104.40	.9579	.5345	72	253.02	.3952	.2205
23	106.86	.9358	.5222	73	256.42	.3900	.2176
24	109.38	.9142	.5101	74	259.82	.3849	.2148
25	111.91	.8937	.4987	75	263.25	.3799	.2120
26	114.45	.8737	.4875	76	266.67	.3750	.2093
27	117.03	.8545	.4768	77	270.12	.3702	.2066
28	119.62	.8360	.4665	78	273.57	.3655	.2040
29	122.25	.8180	.4564	79	277.01	.3610	.2014
30	124.89	.8007	.4468	80	280.53	.3565	.1989
31	127.54	.7840	.4375	81	284.03	.3521	.1965
32	130.22	.7679	.4285	82	287.53	.3478	.1941
33	132.93	.7523	.4198	83	291.03	.3436	.1917
34	135.66	.7371	.4113	84	294.54	.3395	.1894
35	138.40	.7225	.4032	85	298.06	.3355	.1872
36	141.15	.7085	.3953	86	301.63	.3315	.1850
37	143.95	.6947	.3876	87	305.21	.3276	.1828
38	146.76	.6814	.3802	88	308.78	.3239	.1807
39	149.60	.6685	.3730	89	312.35	.3202	.1787
40	152.45	.6560	.3661	90	315.92	.3165	.1766
41	155.30	.6439	.3593	91	319.53	.3130	.1747
42	158.20	.6321	.3527	92	323.13	.3095	.1727
43	161.11	.6207	.3464	93	326.74	.3060	.1707
44	164.02	.6097	.3402	94	330.38	.3027	.1689
45	167.00	.5988	.3341	95	334.01	.2994	.1671
46	169.97	.5883	.3283	96	337.65	.2962	.1653
47	172.95	.5782	.3226	97	341.30	.2930	.1635
48	175.95	.5683	.3171	98	344.96	.2899	.1618
49	178.95	.5588	.3118	99	348.63	.2868	.1600
50	182.00	.5494	.3066	100	352.30	.2838	.1584

* The viscosity of water at 20.20° C is 1.0000 centipoise.

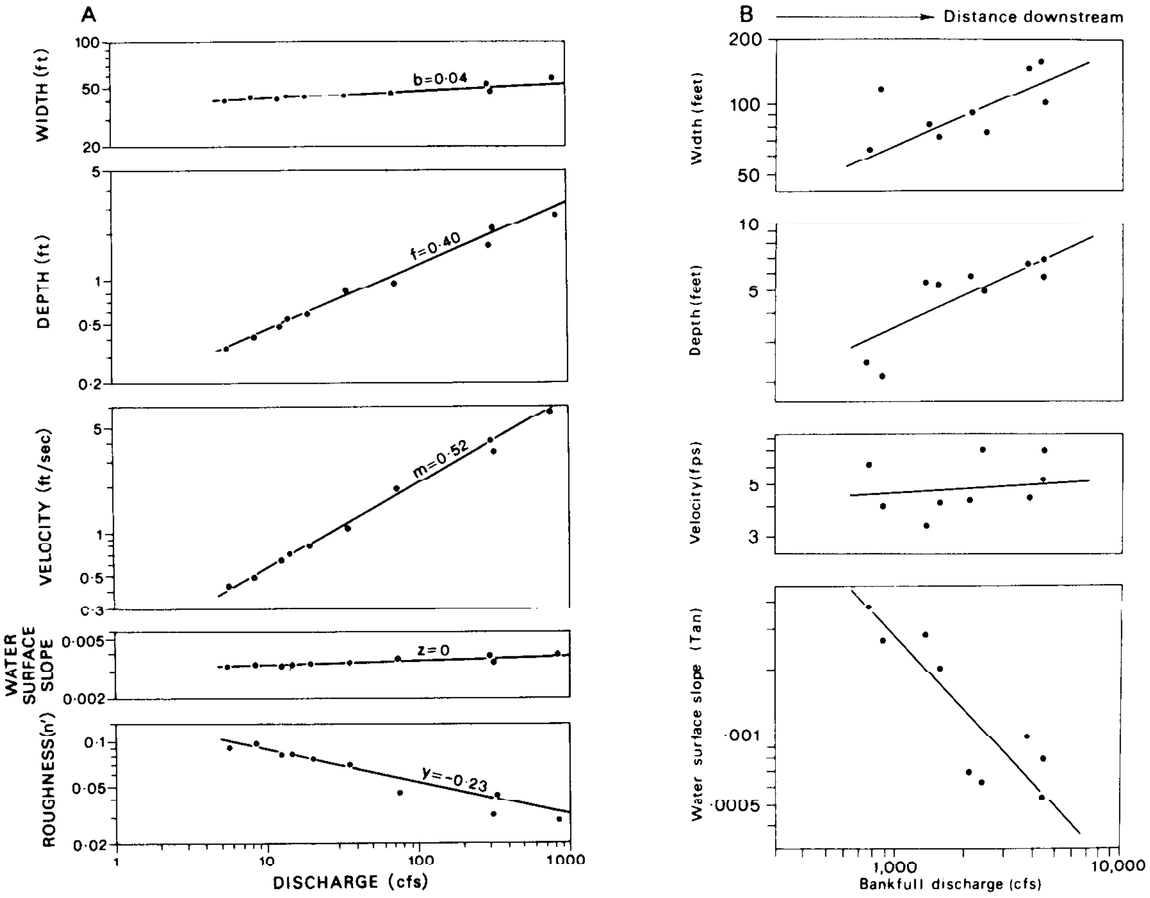


Figure 12.18 A. Relation of width, mean depth, mean velocity, water surface slope and roughness parameter (*n'*) to changing discharge at-a-station from Brandywine Creek at Cornog, Pennsylvania. B. Relation of width, mean depth, mean velocity and water surface slope to bankfull discharge along a 40-km (25-mile) stretch of Brandywine Creek, Pennsylvania.
Source: Wolman, 1955, figures 10 and 29, pp. 11, 31.

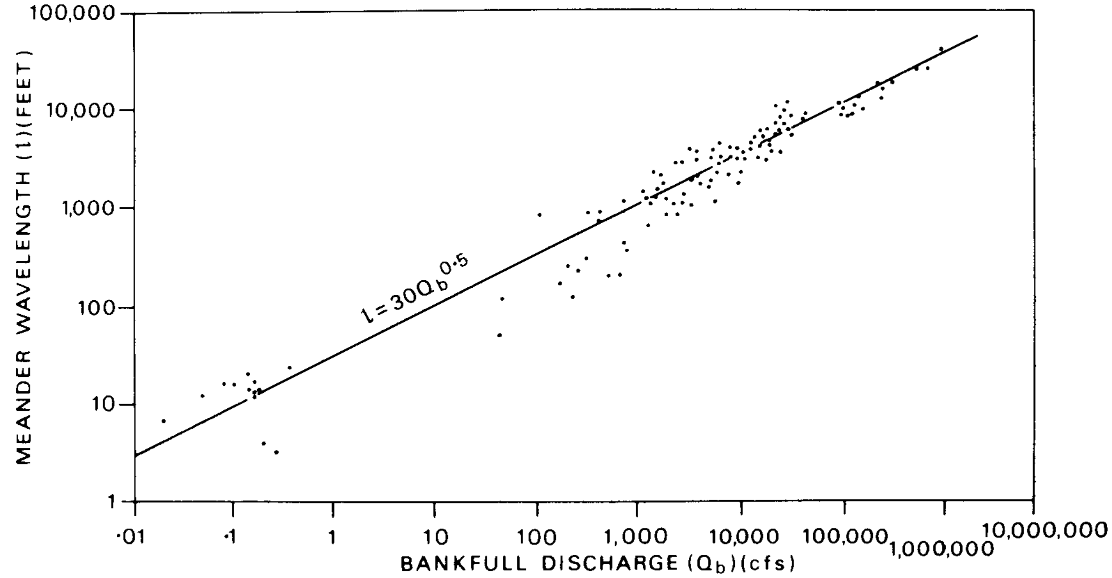


Figure 12.19 Relation between meander wavelength and bankfull discharge