Bedrock Channels: Incision Rates and Longitudinal Profiles

Bedrock Channels are actively incising into rock. Incision rate is set by the ability of flows (and sediment tools carried by the flows) to abrade or “detach” bedrock. In this way they are distinct from transport-limited channels, though in many mixed bedrock-alluvial channels (which are common), this distinction can be blurred.

Transport capacity: $Q_c$
Sediment Supply (Flux): $Q_s$

$Q_s / Q_c \to$ very small

Erosion is governed by ability to “detach” or incision into bedrock, not limited by $\frac{\partial q_s}{\partial x}$.

Therefore, erosion is highest where shear stress is highest, rather than where it is increasing most rapidly. See Whipple, 2004, Annual Reviews in Earth and Planetary Sciences, review paper for background and details.

A. Derivation of a Simple (Generic – not process-specific)

Detachment-Limited Incision Model

Shear Stress model: erosion proportional to shear stress to a power (all below directly analogous in case of unit stream power):

$E = k_b(\tau_b^a - \tau_c^a)$ or $E = k_b(\tau_b - \tau_c)^a$; $k_b = k_e f(q_s)$

$k_e$ is erosivity at optimum sediment load.
Conservation of Mass (water):

\[ Q = \bar{u}hW \]

Conservation of Momentum (steady uniform flow):

\[ \tau_b = \rho g h S \]
\[ \tau_b = \rho C_f \bar{u}^2 \]

Goal: write \( \tau_b \) in terms of slope, discharge.

Use conservation of mass (water), substitute into friction relation:

\[ \tau_b = \rho C_f Q^2 (Wh)^{-2} \]

Solve shear stress equation for flow depth, substitute into above gives:

\[ \tau_b^3 = \rho^3 g^2 C_f Q^2 (W)^{-2} S^2 \]
\[ \tau_b = \rho g^{2/3} C_f^{1/3} (Q/W)^{2/3} S^{2/3} \]

Can be written as:

\[ \tau_b = k_1 (Q/W)^\alpha S^\beta \]

where \( k_1 = \rho g^\alpha C_f^{\alpha/2} \); \( \alpha = \frac{2}{3} \), \( \beta = \frac{2}{3} \) (Generalized Darcy-Weisbach friction relation);

\( \alpha = \frac{3}{5} \), \( \beta = \frac{2}{10} \) (Manning)

Channel Width Closure

Empirical relation for hydraulic geometry (channel width closure used if direct measurements of \( W \) not available):

\( W = k_w Q^b \); \( b \approx 0.5 \) typical in both alluvial and bedrock rivers.

Substitute into relation for shear stress:

\[ \tau_b = k_1 k_w^{-\alpha} Q^{\alpha(1-b)} S^\beta \]

Drainage Basin Hydrology

For application to ungauged rivers, use empirical relation for basin hydrology:

\[ Q = k_q A^c \]

0.7 ≤ \( c \) ≤ 1.0 typical (\( c < 1 \) reflects: storm size < basin size, short storm duration, non-uniform ppt, groundwater losses, storage in floodplains, etc)
Combine Above to Derive the Stream Power Incision Model

Substitute into relation for shear stress:

$$\tau_b = k_i k_w a^a k_q a^{(1-b)} A^{a(1-b)} S^\beta$$

Combine these for case $\tau_c << \tau_b$ in floods of interest $E = k_c f(q_s) \tau_b^a$ gives the well-known “Stream Power River Incision Model”:

$$E = KA^n S^n$$

$$n = \beta a ; m = aac(1-b)$$

$$K = k_c f(q_s) k_w a^a k_q a^{(1-b)} k_i a$$

$$\frac{m}{n} = \frac{\alpha}{\beta} c(1-b) ; \frac{\alpha}{\beta} = 1$$ for Generalized Darcy-Weisbach friction relation

$m/n \sim 0.5$ characteristic of broad range of fluvial incision processes that scale with shear stress (or unit stream power) raise to some power ($a$). If erosion process is linear in shear stress ($a = 1$), expected exponents in the stream power incision model are:

$m \sim 1/3, n \sim 2/3.$

Empirical Field Support


$$\frac{dz}{dt} = 0.11 A^{0.44} S^{0.68} ; R^2 = .85 (50 \text{ data points})$$

95% confidence intervals: $0.06 < K < .21; .38 < m < .51; .58 < n < .78$
**B. Conservation of Mass (Rock): Profile Evolution**

Now we can consider conservation of mass of the rock to write an evolution equation for a bedrock channel:

\[
\frac{\partial z}{\partial t} = U - E = U - KA^n S^n
\]

At **steady state**, \( U = E \) such that \( \frac{\partial z}{\partial t} = 0 \), such that we can write:

\[
U = KA^n S^n
\]

this can be solved directly for the steady-state river slope:

\[
S = \left( \frac{U}{K} \right)^\frac{1}{n} A^{-\frac{m}{n}}
\]

Thus a power-law relation between local channel slope and upstream drainage area is predicted: a straight line in plot of \( \log S \) vs. \( \log A \) with slope \(-m/n\) (concavity index) and intercept \((U/K)^{1/n}\) (steepness index) (this is true only if \( U, K, m, \) and \( n \) are spatially uniform … what might happen to profile concavity where \( K = K(q_s) = K(x) \)?)